THE ACCELERATED PLACING OF A GYROSCOPIC COMPASS IN A MERIDIAN

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The gyroscopic compass represents an instrument with a large period of natural vibrations, of the order of one hour and a half. For the damping of the natural vibrations which may occur because the gyrocompass in starting was not along a meridian, a considerable time will be required, equal to three to four periods of natural vibrations. There is an interest, therefore, to study methods of an accelerated placing of the gyrocompass in the meridian. Usually, the latter is accomplished by an application to the gyrocompass of additional external forces. The problem regarding the selection of the law of control of these forces is discussed below.

1. The equations of motion of a gyrocompass fitted with a hydraulic damper for the damping of natural vibrations, are of the following form [1]:

$$\begin{gathered} A\alpha^{\cdot\cdot} + H\beta^{\cdot} + HU\cos\varphi\alpha = 0 \\ B\beta^{\cdot\cdot} - H\alpha^{\cdot} + lP\beta + lP(1-\rho)\vartheta = HU\sin\varphi + Q(t) \\ \vartheta^{\cdot} + F\vartheta + F\beta = 0 \end{gathered} \tag{1.1}$$

Here α is the angle of rotation of the gyrocompass in the aximuth, β is the angle of elevation of the Northern diameter of the gyrosphere above the horizontal plane, θ is the angle of inclination of the fluid plane in the hydraulic damper above the equator plane of the gyrosphere. H indicates the resultant kinetic moment of the gyroscope, mounted in the gyrosphere, LP is the static moment of the gyrosphere, A and B are moments of inertia of the gyrosphere with respect to the corresponding axes, U is the angular velocity of the daily rotation of the earth, ϕ is the latitude of the place of observation.

We indicate by $Q(t)$ the additional generalized external force which represents a moment with respect to the eastern diameter of the gyrosphere, applied for the purpose of an accelerated placing of the gyroscope into a meridian. The law of time dependence of this external force is to be determined. Limiting ourselves to the study of the precessional motion of the gyrocompass, we omit the inertia terms $A\alpha''$ and $B\beta''$ in equation (1.1). Equation (1.1) then is reduced to the form:

$$y_1^{\cdot} - \frac{k^2}{U\cos\varphi} y_2 - \frac{k^2}{U\cos\varphi}(1-\rho) y_3 = q_1(t) \tag{1.2}$$

$$y_2^{\cdot} + U\cos\varphi y_1 = 0, \qquad y_3^{\cdot} + F y_3 + F y_2 = 0 \tag{1.3}$$

where

$$y_1 = \alpha, \quad y_2 = \beta - \frac{HU\sin\varphi}{\rho l P}, \quad y_3 = \vartheta + \frac{HU\sin\varphi}{\rho l P}, \quad k^2 = \frac{lPU\cos\varphi}{H}, \quad q_1(t) = -\frac{Q(t)}{H}$$

The characteristic determinant of the system of equations (1.2) is of the form

$$\Delta(D) = D^3 + FD^2 + k^2 D + \rho k^2 F \tag{1.4}$$

The stability condition of the gyrocompass, as seen from (1.4 will be the following:

$$0 < \rho < 1 \tag{1.5}$$

The parameters of the gyrocompass are usually selected in such a manner that the characteristic equation

$$\Delta(D) = 0 \tag{1.6}$$

has a real root D_1 and two complex conjugate roots D_2, D_3. Designating these roots by

$$D_1 = \varkappa, \qquad D_2, D_3 = \varepsilon \pm i\omega \tag{1.7}$$

we will have

$$\Delta(D) = (D - \varkappa)(D - \varepsilon - i\omega)(D - \varepsilon + i\omega) \tag{1.8}$$

The solution of the equations (1.2) is of the form:

$$y_i(t) = A_i e^{\varkappa t} + B_i e^{\varepsilon t}\cos\omega t - C_i e^{\varepsilon t}\sin\omega t + \int_0^t N_{i1}(t-u)\, q_1(u)\, du \qquad (i = 1,2,3) \tag{1.9}$$

Here

$$\begin{aligned} A_i &= a_{i1} y_1(0) + a_{i2} y_2(0) + a_{i3} y_3(0) \\ B_i &= b_{i1} y_1(0) + b_{i2} y_2(0) + b_{i3} y_3(0) \\ C_i &= c_{i1} y_1(0) + c_{i2} y_2(0) + c_{i3} y_3(0) \end{aligned} \tag{1.10}$$

$$N_{i1}(t-u) = a_{i1} e^{\varkappa(t-u)} + b_{i1} e^{\varepsilon(t-u)}\cos\omega(t-u) - c_{i1} e^{\varepsilon(t-u)}\sin\omega(t-u) \tag{1.11}$$

and the coefficients a_{ij}, b_{ij}, c_{ij} are determined by the expressions

$$a_{11} = \mu\varkappa(\varkappa + F), \qquad a_{21} = -\mu(\varkappa + F)U\cos\varphi, \qquad a_{31} = \mu FU\cos\varphi \tag{1.12}$$

$$a_{12} = \mu\frac{k^2}{U\cos\varphi}(\varkappa + \rho F), \qquad a_{22} = \mu\varkappa(\varkappa + F), \qquad a_{32} = -\mu\varkappa F$$

$$a_{13} = \mu\frac{k^2}{U\cos\varphi}(1-\rho)\varkappa, \qquad a_{23} = -\mu k^2(1-\rho), \qquad a_{33} = \mu(\varkappa^2 + k^2)$$

$$b_{11} = \mu[\varepsilon^2 + \omega^2 - \varkappa(F + 2\varepsilon)], \quad b_{21} = \mu(\varkappa + F)U\cos\varphi, \qquad b_{31} = -\mu FU\cos\varphi$$

$$b_{12} = -\mu\frac{k^2}{U\cos\varphi}(\varkappa + \rho F), \quad b_{22} = \mu[\varepsilon^2 + \omega^2 - \varkappa(F + 2\varepsilon)], \; b_{32} = \mu\varkappa F$$

$$b_{13} = -\mu\varkappa\frac{k^2}{U\cos\varphi}(1-\rho), \quad b_{23} = \mu k^2(1-\rho), \qquad b_{33} = \mu(\varepsilon^2 + \omega^2 - k^2 - 2\varepsilon\varkappa)$$

$$c_{11} = -\frac{\mu}{\omega}[\varepsilon^3 + (F - \varkappa)\varepsilon^2 + (\omega^2 - \varkappa F)\varepsilon + \omega^2(\varkappa + F)]$$

$$c_{12} = -\frac{\mu}{\omega}\frac{k^2}{U\cos\varphi}[(\varepsilon - \varkappa)(\varepsilon + \rho F) + \omega^2], \quad c_{23} = \frac{\mu}{\omega}k^2(1-\rho)(\varepsilon - \varkappa)$$

$$c_{13} = -\frac{\mu}{\omega}\frac{k^2}{U\cos\varphi}(1-\rho)[\varepsilon(\varepsilon - \varkappa) + \omega^2], \quad c_{31} = -\frac{\mu}{\omega}(\varepsilon - \varkappa)FU\cos\varphi$$

$$c_{21} = \frac{\mu}{\omega}U\cos\varphi[(\varepsilon - \varkappa)(\varepsilon + F) + \omega^2], \qquad c_{32} = \frac{\mu}{\omega}[\varepsilon F(\varepsilon - \varkappa) + \omega^2 F]$$

$$c_{22} = -\frac{\mu}{\omega}[\varepsilon^3 + (F - \varkappa)\varepsilon^2 + (\omega^2 - \varkappa F)\varepsilon + \omega^2(\varkappa + F)],$$

$$c_{33} = \frac{\mu}{\omega}[-\varepsilon^3 + \varkappa\varepsilon^2 - (k^2 + \omega^2)\varepsilon + \varkappa(k^2 - \omega^2)] \qquad \left(\mu = \frac{1}{(\varkappa - \varepsilon)^2 + \omega^2}\right)$$

2. We pass now to a selection of the law of variation of the function $q_1(t)$ with respect to time, starting from the requirement that at the instant of time $t = T$ the gyrocompass be placed in a meridian [2]. As follows from (1.9), it is necessary for this that

$$\int_0^T N_{i1}(T-u)\,q_1(u)\,du = R_i(T) \qquad (i=1,2,3) \tag{2.1}$$

where

$$R_i(T) = -A_i e^{\varkappa T} - B_i e^{\varepsilon T}\cos\omega T + C_i e^{\varepsilon T}\sin\omega T \tag{2.2}$$

The interval of time $(0, T)$ will be divided into three equal intervals $(0, t_1)$, (t_1, t_2), (t_2, T) and we assume $q_1(t)$ to be a step function which has constant values along these intervals of time. These values shall be designated by $q_1(0)$, $q_1(t_1)$ and $q_1(t_2)$, respectively. Relations (2.1) now take the form

$$c_i^{(0)}q_1(0) + c_i^{(1)}q_1(t_1) + c_i^{(2)}q_1(t_2) = R_i(T) \qquad (i = 1,2,3) \tag{2.3}$$

where

$$c_i^{(0)} = \int_0^{t_1} N_{i1}(T-u)\,du, \qquad c_i^{(1)} = \int_{t_1}^{t_2} N_{i1}(T-u)\,du, \qquad c_i^{(2)} = \int_{t_2}^{T} N_{i1}(T-u)\,du \tag{2.4}$$

Substituting into (2.4) expression (1.11), which determines the functions $N_{i1}(T-u)$, and carrying out the integration, it is possible to represent $c_i^{(0)}$, $c_i^{(1)}$, $c_i^{(2)}$ in the form

$$c_i^{(0)} = \xi_i m_1 + \eta_i n_1 + \zeta_i p_1, \quad c_i^{(1)} = \xi_i m_2 + \eta_i n_2 + \zeta_i p_2, \quad c_i^{(2)} = \xi_i m_3 + \eta_i n_3 + \zeta_i p_3 \tag{2.5}$$

where

$$\begin{gathered}
\xi_i = a_{i1} e^{\varkappa T}, \quad \eta_i = e^{\varepsilon T}(b_{i1}\cos\omega T - c_{i1}\sin\omega T), \qquad \zeta_i = e^{\varepsilon T}(b_{i1}\sin\omega T + c_{i1}\cos\omega T),\\
m_1 = \frac{1}{\varkappa}(1 - e^{-\varkappa t_1}), \quad m_2 = \frac{1}{\varkappa}(e^{-\varkappa t_1} - e^{-\varkappa t_2}), \quad m_3 = \frac{1}{\varkappa}(e^{-\varkappa t_2} - e^{-\varkappa T})\\
n_1 = \frac{-\varepsilon\cos\omega t_1 + \omega\sin\omega t_1}{\varepsilon^2+\omega^2} e^{-\varepsilon t_1} + \frac{\varepsilon}{\varepsilon^2+\omega^2}\\
n_2 = \frac{-\varepsilon\cos\omega t_2 + \omega\sin\omega t_2}{\varepsilon^2+\omega^2} e^{-\varepsilon t_2} - \frac{-\varepsilon\cos\omega t_1 + \omega\sin\omega t_1}{\varepsilon^2+\omega^2} e^{-\varepsilon t_1}\\
n_3 = \frac{-\varepsilon\cos\omega T + \omega\sin\omega T}{\varepsilon^2+\omega^2} e^{-\varepsilon T} - \frac{-\varepsilon\cos\omega t_2 + \omega\sin\omega t_2}{\varepsilon^2+\omega^2} e^{-\varepsilon t_2}\\
p_1 = \frac{-\varepsilon\sin\omega t_1 - \omega\cos\omega t_1}{\varepsilon^2+\omega^2} e^{-\varepsilon t_1} + \frac{\omega}{\varepsilon^2+\omega^2}\\
p_2 = \frac{-\varepsilon\sin\omega t_2 - \omega\cos\omega t_2}{\varepsilon^2+\omega^2} e^{-\varepsilon t_2} + \frac{\varepsilon\sin\omega t_1 + \omega\cos\omega t_1}{\varepsilon^2+\omega^2} e^{-\varepsilon t_1}\\
p_3 = \frac{-\varepsilon\sin\omega T - \omega\cos\omega T}{\varepsilon^2+\omega^2} e^{-\varepsilon T} + \frac{\varepsilon\sin\omega t_2 + \omega\cos\omega t_2}{\varepsilon^2+\omega^2} e^{-\varepsilon t_2}
\end{gathered} \tag{2.6}$$

From equations (2.3) we obtain

$$q_1(0) = \frac{\Delta_1}{\Delta}, \qquad q_1(t_1) = \frac{\Delta_2}{\Delta}, \qquad q_1(t_2) = \frac{\Delta_3}{\Delta} \tag{2.7}$$

where

$$\Delta_1 = \begin{vmatrix} R_1(T) & c_1^{(1)} & c_1^{(2)} \\ R_2(T) & c_2^{(1)} & c_2^{(2)} \\ R_3(T) & c_3^{(1)} & c_3^{(2)} \end{vmatrix} \qquad \Delta_2 = \begin{vmatrix} c_1^{(0)} & R_1(T) & c_1^{(2)} \\ c_2^{(0)} & R_2(T) & c_2^{(2)} \\ c_3^{(0)} & R_3(T) & c_3^{(2)} \end{vmatrix} \tag{2.8}$$

$$\Delta_3 = \begin{vmatrix} c_1^{(0)} & c_1^{(1)} & R_1(T) \\ c_2^{(0)} & c_2^{(1)} & R_2(T) \\ c_3^{(0)} & c_3^{(1)} & R_3(T) \end{vmatrix} \qquad \Delta = \begin{vmatrix} c_1^{(0)} & c_1^{(1)} & c_1^{(2)} \\ c_2^{(0)} & c_2^{(1)} & c_2^{(2)} \\ c_3^{(0)} & c_3^{(1)} & c_3^{(2)} \end{vmatrix}$$

Expressions (2.7) are those which determine the law, in accordance to which $q_1(t)$ must vary in order that the gyrocompass be placed into a meridian at the instant of time $t = T$.

3. As an example let us examine a gyrocompass with the following parameters [1].

$$k^2 = 1.5\cdot 10^{-6}\ \text{сек}^{-2}, \qquad \rho = 0.4, \qquad F = 1.5\cdot 10^{-8}\ \text{sec}^{-1}$$

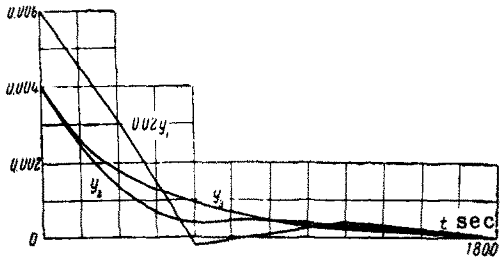
The latitude of the location of observation shall be taken as equal to 60°, such that

$$U \cos\varphi = 3.646 \cdot 10^{-5} \text{ sec}^{-1}.$$

The time interval, during which the gyrocompass should be placed into the meridian, is $T = 1800$ sec. The initial deviations are

$$y_1(0) = 0{,}3, \quad y_2(0) = y_3(0) = 0.004.$$

With these data the values of $q_1(0)$, $q(t)$, $q(t)$ are:



$$q_1(0) = -0.612 \cdot 10^{-3}, \qquad q_1(t_1) = 0.0159 \cdot 10^{-3}, \qquad q_1(t_2) = -0.0412 \cdot 10^{-3} \; [\text{sec}^{-1}]$$

With the value of the resultant kinetic moment $H = 155000$ g*cm sec, the additional generalized external force $Q(t) = -Hq_1(t)$ will have the following values along the intervals of time $(0, t_1)$, (t_1, t_2), (t_2, T), respectively:

$$Q(0) = 94.86, \qquad Q(t_1) = -2.46, \qquad Q(t_2) = 6.38 \; [\text{g*cm}]$$

The process of placing the gyrocompass into the meridian is graphically represented by functions $y_1(t)$, $y_2(t)$, $y_3(t)$ in the figure.

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